

APPENDIX 1

Mathematics

Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Information and communication technology (ICT)

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used.

Spoken language

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

School curriculum

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate. All schools are also required to set out their school curriculum for mathematics on a year-by-year basis and make this information available online.

Attainment targets

By the end of each key stage, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

Key stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources (e.g. concrete objects and measuring tools).

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

Year 1

Year 1 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number▪ count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens▪ given a number, identify one more and one less▪ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least▪ read and write numbers from 1 to 20 in numerals and words.	<p>NUMBER</p> <p>Number and place value</p> <p>Pupils practise counting (1, 2, 3), ordering (e.g. first, second, third), or to indicate a quantity (e.g. 3 apples, 2 centimetres), including solving simple concrete problems, until they are fluent.</p> <p>Pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations.</p> <p>They practise counting as reciting numbers and counting as enumerating objects, and counting in twos, fives and tens from different multiples to develop their recognition of patterns in the number system (e.g. odd and even numbers), including varied and frequent practice through increasingly complex questions.</p> <p>They recognise and create repeating patterns with objects and with shapes.</p>
<p>Addition and subtraction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	<p>Addition and subtraction</p> <p>Pupils memorise and reason with number bonds to 10 and 20 in several forms (e.g. $9 + 7 = 16$; $16 - 7 = 9$; $7 = 16 - 9$). They should realise the effect of adding or subtracting zero. This establishes</p>

Year 1 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> represent and use number bonds and related subtraction facts within 20 add and subtract one-digit and two-digit numbers to 20, including zero solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$. 	<p>addition and subtraction as related operations.</p> <p>Pupils combine and increase numbers, counting forwards and backwards.</p> <p>They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms put together, add, altogether, total, take away, distance between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.</p>
<p>Multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. 	<p>Multiplication and division</p> <p>Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.</p> <p>They make connections between arrays, number patterns, and counting in twos, fives and tens.</p>
<p>Fractions</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recognise, find and name a half as one of two equal parts of an object, shape or quantity recognise, find and name a quarter as one of four equal 	<p>Fractions</p> <p>Pupils are taught half and quarter as ‘fractions of’ discrete and continuous quantities by solving problems using shapes, objects and quantities. For example, they could recognise and find half a length, quantity, set of objects or shape. Pupils connect halves and quarters to the equal sharing and grouping of sets of objects and to</p>

Year 1 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>parts of an object, shape or quantity.</p>	<p>measures, as well as recognising and combining halves and quarters as parts of a whole.</p>
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ compare, describe and solve practical problems for: <ul style="list-style-type: none"> ▪ lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half) ▪ mass or weight (e.g. heavy/light, heavier than, lighter than) ▪ capacity/volume (full/empty, more than, less than, quarter) ▪ time (quicker, slower, earlier, later) ▪ measure and begin to record the following: <ul style="list-style-type: none"> ▪ lengths and heights ▪ mass/weight ▪ capacity and volume ▪ time (hours, minutes, seconds) ▪ recognise and know the value of different denominations of coins and notes ▪ sequence events in chronological order using language such as: before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening 	<p>MEASUREMENT</p> <p>The pairs of terms: mass and weight, volume and capacity, are used interchangeably at this stage.</p> <p>Pupils move from using and comparing different types of quantities and measures using non-standard units, including discrete (e.g. counting) and continuous (e.g. liquid) measurement, to using manageable common standard units.</p> <p>In order to become familiar with standard measures, pupils begin to use measuring tools such as a ruler, weighing scales and containers.</p> <p>Pupils use the language of time, including telling the time throughout the day, first using o'clock and then half past.</p>

Year 1 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> recognise and use language relating to dates, including days of the week, weeks, months and years tell the time to the hour and half past the hour and draw the hands on a clock face to show these times. 	
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recognise and name common 2-D and 3-D shapes, including: <ul style="list-style-type: none"> 2-D shapes (e.g. rectangles (including squares), circles and triangles) 3-D shapes (e.g. cuboids (including cubes), pyramids and spheres). 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils handle common 2-D and 3-D shapes, naming these and related everyday objects fluently. They recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids can be different shapes.</p>
<p>Position and direction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> describe position, directions and movements, including half, quarter and three-quarter turns. 	<p>Position and direction</p> <p>They use the language of position, direction and motion, including: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside.</p> <p>Pupils make half, quarter and three-quarter turns and routinely make these turns in a clockwise direction.</p>

Year 2

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward▪ recognise the place value of each digit in a two-digit number (tens, ones)▪ identify, represent and estimate numbers using different representations, including the number line▪ compare and order numbers from 0 up to 100; use $<$, $>$ and $=$ signs▪ read and write numbers to at least 100 in numerals and in words▪ use place value and number facts to solve problems.	<p>NUMBER</p> <p>Number and place value</p> <p>Using materials and a range of representations, pupils practise counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. They count in multiples of three to support their later understanding of a third.</p> <p>As they become more confident with numbers up to 100, pupils are introduced to larger numbers to develop further their recognition of patterns within the number system and represent them in different ways, including spatial representations.</p> <p>Pupils should partition numbers in different ways (e.g. $23 = 20 + 3$ and $23 = 10 + 13$) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.</p>
<p>Addition and subtraction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ solve problems with addition and subtraction:	<p>Addition and subtraction</p> <p>Pupils extend their understanding of the language of addition and subtraction to include sum and difference.</p>

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
<ul style="list-style-type: none"> ▪ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ▪ applying their increasing knowledge of mental and written methods ▪ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ▪ add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> ▪ a two-digit number and ones ▪ a two-digit number and tens ▪ two two-digit numbers ▪ adding three one-digit numbers ▪ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot ▪ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. 	<p>Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using $3 + 7 = 10$, $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$, $100 - 70 = 30$ and $70 = 100 - 30$. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). This establishes commutativity and associativity of addition.</p> <p>Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.</p>
<p>Multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ recall and use multiplication and division facts for the 2, 5 	<p>Multiplication and division</p> <p>Pupils use a variety of language to describe multiplication and division.</p>

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
<p>and 10 multiplication tables, including recognising odd and even numbers</p> <ul style="list-style-type: none"> calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. 	<p>Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.</p> <p>Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, and relating these to fractions and measures (e.g. $40 \div 2 = 20$, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (e.g. $4 \times 5 = 20$ and $20 \div 5 = 4$).</p>
<p>Fractions</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity write simple fractions e.g. $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$. 	<p>Fractions</p> <p>Pupils use additional fractions as ‘fractions of’ discrete and continuous quantities by solving problems using shapes, objects and quantities. They connect unit fractions to equal sharing and grouping, to numbers when they can be calculated, and to measures, finding fractions of lengths, quantity, a set of objects or shapes. They meet $\frac{3}{4}$ as the first example of a non-unit fraction.</p> <p>Pupils should count in fractions up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line (e.g. $1\frac{1}{4}$, $1\frac{2}{4}$ (or $1\frac{1}{2}$), $1\frac{3}{4}$, 2). This reinforces the concept of fractions as</p>

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
	numbers and that they can add up to more than one.
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels compare and order lengths, mass, volume/capacity and record the results using >, < and = recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value find different combinations of coins that equal the same amounts of money solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change compare and sequence intervals of time tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times. 	<p>MEASUREMENT</p> <p>Pupils use standard units of measurement with increasing accuracy, using their knowledge of the number system. They use the appropriate language and record using standard abbreviations.</p> <p>They become fluent in telling the time on analogue clocks and recording it.</p> <p>Pupils become fluent in counting and recognising coins. They read and say amounts of money confidently and use the symbols £ and p accurately, recording pounds and pence separately.</p>

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> identify and describe the properties of 2-D shapes, including the number of sides and symmetry in a vertical line identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces identify 2-D shapes on the surface of 3-D shapes, for example a circle on a cylinder and a triangle on a pyramid compare and sort common 2-D and 3-D shapes and everyday objects. 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils handle and name a wider variety of common 2-D and 3-D shapes including: quadrilaterals and cuboids, prisms, cones and polygons, and identify the properties of each shape (e.g. number of sides, number of faces). Pupils identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces.</p> <p>Pupils read and write names for shapes that are appropriate for their word reading and spelling.</p> <p>Pupils draw lines and shapes using a straight edge.</p>
<p>Position and direction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> order and arrange combinations of mathematical objects in patterns use mathematical vocabulary to describe position, direction and movement including distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise), and movement in a straight line. 	<p>Position and direction</p> <p>Pupils should work with patterns of shapes, including those in different orientations.</p> <p>Pupils use the concept and language of angles to describe ‘turn’ by applying rotations, including in practical contexts (e.g. pupils themselves moving in turns, giving instructions to other pupils to do so, and programming robots using instructions given in right angles).</p>

Year 2 programme of study (statutory requirements)	Notes and Guidance (non-statutory)
<p>STATISTICS</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ interpret and construct simple pictograms, tally charts, block diagrams and simple tables ▪ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ▪ ask and answer questions about totalling and comparing categorical data. 	<p>STATISTICS</p> <p>Pupils record, interpret, collate, organise and compare information (e.g. using many-to-one correspondence with simple ratios 2, 5, 10).</p>

Lower key stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

Year 3

Year 3 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number▪ recognise the place value of each digit in a three-digit number (hundreds, tens, ones)▪ compare and order numbers up to 1000▪ identify, represent and estimate numbers using different representations▪ read and write numbers up to 1000 in numerals and in words▪ solve number problems and practical problems involving these ideas.	<p>NUMBER</p> <p>Number and place value</p> <p>Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.</p> <p>They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (e.g. $146 = 100$ and 40 and 6, $146 = 130$ and 16).</p> <p>Using a variety of representations, including those related to measure, pupils continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.</p>
<p>Addition and subtraction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ add and subtract numbers mentally, including:<ul style="list-style-type: none">▪ a three-digit number and ones▪ a three-digit number and tens	<p>Addition and subtraction</p> <p>Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.</p> <p>Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly</p>

Year 3 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ a three-digit number and hundreds ▪ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ▪ estimate the answer to a calculation and use inverse operations to check answers ▪ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction. 	<p>large numbers up to three digits to become fluent (see Appendix 1).</p>
<p>Multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ▪ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ▪ solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects. 	<p>Multiplication and division</p> <p>Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.</p> <p>Pupils develop efficient mental methods, for example, using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).</p> <p>Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.</p>

Year 3 programme of study (statutory requirements)	Notes and guidance (non-statutory)
	<p>Pupils solve simple problems in contexts, deciding which of the four operations to use and why, including measuring and scaling contexts, and correspondence problems in which m objects are connected to n objects (e.g. 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).</p>
<p>Fractions</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ▪ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators ▪ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators ▪ recognise and show, using diagrams, equivalent fractions with small denominators ▪ add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$) ▪ compare and order unit fractions, and fractions with the same denominators ▪ solve problems that involve all of the above. 	<p>Fractions</p> <p>Pupils connect tenths to place value, decimal measures and to division by 10.</p> <p>They begin to understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence. They should go beyond the [0, 1] interval, relating this to measure.</p> <p>Pupils understand the relation between unit fractions as operators (fractions of), and division by integers.</p> <p>They continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, or unit fractions as a division of a quantity.</p> <p>Pupils practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.</p>

Year 3 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) ▪ measure the perimeter of simple 2-D shapes ▪ add and subtract amounts of money to give change, using both £ and p in practical contexts ▪ tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks ▪ estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight ▪ know the number of seconds in a minute and the number of days in each month, year and leap year ▪ compare durations of events, for example to calculate the time taken by particular events or tasks. 	<p>MEASUREMENT</p> <p>Pupils continue to measure using the appropriate tools and units, progressing to using a wider range of measures, including comparing and using mixed units (e.g. 1 kg and 200g) and simple equivalents of mixed units (e.g. 5m = 500cm).</p> <p>The comparison of measures should also include simple scaling by integers (e.g. a given quantity or measure is twice as long or five times as high) and this should connect to multiplication.</p> <p>Pupils continue to become fluent in recognising the value of coins, by adding and subtracting amounts, including mixed units, and giving change using manageable amounts. They record £ and p separately. The decimal recording of money is introduced formally in year 4.</p> <p>Pupils use both analogue and digital 12-hour clocks and record their times. In this way they become fluent in and prepared for using digital 24-hour clocks in year 4.</p>

Year 3 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them recognise that angles are a property of shape or a description of a turn identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle identify horizontal and vertical lines and pairs of perpendicular and parallel lines. 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils' knowledge of the properties of shapes is extended at this stage to symmetrical and non-symmetrical polygons and polyhedra. Pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle.</p> <p>Pupils should draw and measure straight lines in centimetres.</p>
<p>STATISTICS</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> interpret and present data using bar charts, pictograms and tables solve one-step and two-step questions such as 'How many more?' and 'How many fewer?' using information presented in scaled bar charts and pictograms and tables. 	<p>STATISTICS</p> <p>Pupils understand and use simple scales (e.g. 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy.</p> <p>They continue to interpret data presented in many contexts.</p>

Year 4

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to</p> <ul style="list-style-type: none">▪ count in multiples of 6, 7, 9, 25 and 1000▪ find 1000 more or less than a given number▪ count backwards through zero to include negative numbers▪ recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)▪ order and compare numbers beyond 1000▪ identify, represent and estimate numbers using different representations▪ round any number to the nearest 10, 100 or 1000▪ solve number and practical problems that involve all of the above and with increasingly large positive numbers▪ read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.	<p>NUMBER</p> <p>Number and place value</p> <p>Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice.</p> <p>They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.</p> <p>They connect estimation and rounding numbers to the use of measuring instruments.</p> <p>Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.</p>
<p>Addition and subtraction</p>	<p>Addition and subtraction</p> <p>Pupils continue to practise both mental methods and columnar</p>

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. 	<p>addition and subtraction with increasingly large numbers to aid fluency (see Appendix 1).</p>
<p>Multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers recognise and use factor pairs and commutativity in mental calculations multiply two-digit and three-digit numbers by a one-digit number using formal written layout solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by 	<p>Multiplication and division</p> <p>Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.</p> <p>Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ into $600 \div 3 = 200$.</p> <p>Pupils practise to become fluent in the formal written method of short multiplication for multiplying using multi-digit numbers, and short division with exact answers when dividing by a one-digit number (see Appendix 1).</p> <p>Pupils write statements about the equality of expressions (e.g. use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$). They combine their knowledge of number</p>

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.</p>	<p>facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$.</p> <p>Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.</p>
<p>Fractions (including decimals)</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recognise and show, using diagrams, families of common equivalent fractions count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten. solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number add and subtract fractions with the same denominator recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$ 	<p>Fractions (including decimals)</p> <p>Pupils should connect hundredths to tenths and place value and decimal measure.</p> <p>They extend the use of the number line to connect fractions, numbers and measures.</p> <p>Pupils understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths</p> <p>Pupils make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities. Pupils use factors and multiples to recognise equivalent fractions and simplify where appropriate (e.g. $\frac{6}{9} = \frac{2}{3}$ or $\frac{1}{4} = \frac{2}{8}$).</p> <p>Pupils continue practice in adding and subtracting fractions with the same denominator, to become fluent through a variety of</p>

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths ▪ round decimals with one decimal place to the nearest whole number ▪ compare numbers with the same number of decimal places up to two decimal places ▪ solve simple measure and money problems involving fractions and decimals to two decimal places. 	<p>increasingly complex problems beyond one whole.</p> <p>Pupils are taught throughout that decimals and fractions are different ways of expressing numbers and proportions.</p> <p>Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. This includes relating the decimal notation to division of whole number by 10 and later 100.</p> <p>They practise counting using simple fractions and decimal fractions, both forwards and backwards.</p> <p>Pupils learn decimal notation and the language associated with it, including in the context of measurements. They make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places. They should be able to represent numbers with one or two decimal places in several ways, such as on number lines.</p>
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ Convert between different units of measure (e.g. kilometre to metre; hour to minute) ▪ measure and calculate the perimeter of a rectilinear figure 	<p>MEASUREMENT</p> <p>Pupils build on their understanding of place value and decimal notation to record metric measures, including money.</p> <p>They use multiplication to convert from larger to smaller units.</p> <p>Perimeter can be expressed algebraically as $2(a + b)$ where a and</p>

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>(including squares) in centimetres and metres</p> <ul style="list-style-type: none"> ▪ find the area of rectilinear shapes by counting squares ▪ estimate, compare and calculate different measures, including money in pounds and pence ▪ read, write and convert time between analogue and digital 12 and 24-hour clocks ▪ solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days. 	<p>b are the dimensions in the same unit.</p> <p>They relate area to arrays and multiplication.</p>
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes ▪ identify acute and obtuse angles and compare and order angles up to two right angles by size ▪ identify lines of symmetry in 2-D shapes presented in different orientations ▪ complete a simple symmetric figure with respect to a specific line of symmetry. 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (e.g. isosceles, equilateral, scalene) and quadrilaterals (e.g. parallelogram, rhombus, trapezium).</p> <p>Pupils compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular.</p> <p>Pupils draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams, including where the line of symmetry does not dissect the reflected shape.</p>

Year 4 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>Position and direction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> describe positions on a 2-D grid as coordinates in the first quadrant describe movements between positions as translations of a given unit to the left/right and up/down plot specified points and draw sides to complete a given polygon. 	<p>Position and direction</p> <p>Pupils draw a pair of axes in one quadrant, with equal scales and integer labels. They read, write and use pairs of coordinates (2, 5) including using coordinate-plotting ICT tools.</p>
<p>STATISTICS</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs. 	<p>STATISTICS</p> <p>Pupils understand and use a greater range of scales in their representations.</p> <p>Pupils begin to relate the graphical representation of data to recording change over time.</p>

Upper key stage 2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Year 5

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit▪ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000▪ interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero▪ round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000▪ solve number problems and practical problems that involve all of the above▪ read Roman numerals to 1000 (M) and recognise years written in Roman numerals.	<p>NUMBER</p> <p>Number and place value</p> <p>Pupils identify the place value in large whole numbers.</p> <p>They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far.</p> <p>They should recognise and describe linear number sequences, including those involving fractions and decimals, and find the term-to-term rule.</p>
<p>Addition and subtraction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ add and subtract whole numbers with more than 4 digits,	<p>Addition and subtraction</p> <p>Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>including using formal written methods (columnar addition and subtraction)</p> <ul style="list-style-type: none"> add and subtract numbers mentally with increasingly large numbers use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. 	<p>fluency (see Appendix 1).</p> <p>They practise mental calculations with increasingly large numbers to aid fluency (e.g. $12\,462 - 2\,300 = 10\,162$).</p>
<p>Multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers 	<p>Multiplication and division</p> <p>Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Appendix 1). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.</p> <p>They use and understand the terms factor, multiple and prime, square and cube numbers.</p> <p>Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).</p> <p>Pupils use multiplication and division as inverses to support the</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ multiply and divide numbers mentally drawing upon known facts ▪ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ▪ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 ▪ recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³) ▪ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ▪ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. 	<p>introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.</p> <p>Distributivity can be expressed as $a(b + c) = ab + ac$ in preparation for using algebra.</p>
<p>Fractions (including decimals and percentages)</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ compare and order fractions whose denominators are all multiples of the same number ▪ identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths ▪ recognise mixed numbers and improper fractions and 	<p>Fractions (including decimals and percentages)</p> <p>Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.</p> <p>They extend their knowledge of fractions to thousandths and connect to decimals and measures.</p> <p>Pupils connect equivalent fractions > 1 that simplify to integers with division and fractions > 1 to division with remainders, using the</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>convert from one form to the other and write mathematical statements > 1 as a mixed number (e.g. $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$)</p> <ul style="list-style-type: none"> ▪ add and subtract fractions with the same denominator and multiples of the same number ▪ multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams ▪ read and write decimal numbers as fractions (e.g. $0.71 = \frac{71}{100}$) ▪ recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents ▪ round decimals with two decimal places to the nearest whole number and to one decimal place ▪ read, write, order and compare numbers with up to three decimal places ▪ solve problems involving number up to three decimal places ▪ recognise the per cent symbol (%) and understand that per cent relates to “number of parts per hundred”, and write percentages as a fraction with denominator hundred, and as a decimal fraction ▪ solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those with a denominator of a multiple of 10 or 25. 	<p>number line and other models, and hence move from these to improper and mixed fractions.</p> <p>Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1.</p> <p>Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number.</p> <p>Pupils read and write proper fractions and mixed numbers accurately and continue to practise counting forwards and backwards in simple fractions.</p> <p>Pupils continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities, writing remainders as a fraction.</p> <p>Pupils extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line.</p> <p>Pupils say, read and write decimal fractions and related tenths, hundredths and thousandths accurately and are confident in checking the reasonableness of their answers to problems.</p> <p>They mentally add and subtract tenths, and one-digit whole</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
	<p>numbers and tenths.</p> <p>They practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 (e.g. $0.83 + 0.17 = 1$).</p> <p>Pupils should go beyond the measurement and money models of decimals, for example by solving puzzles involving decimals.</p> <p>Pupils should make connections between percentages, fractions and decimals (e.g. 100% represents a whole quantity and 1% is $1/100$, 50% is $50/100$, 25% is $25/100$) and relate this to finding 'fractions of'. They recognise that percentages are proportions of quantities as well as operators on quantities.</p>
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> convert between different units of metric measure (e.g. kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) understand and use equivalences between metric units and common imperial units such as inches, pounds and pints measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres calculate and compare the area of squares and rectangles 	<p>MEASUREMENT</p> <p>Pupils use their knowledge of place value and multiplication and division to convert between standard units.</p> <p>Pupils calculate the perimeter of rectangles and related composite shapes, including using the relations of perimeter or area to find unknown lengths. Missing measures questions such as these can be expressed algebraically $4 + 2b = 20$ for a rectangle of sides 2 cm and b cm and perimeter of 20cm.</p> <p>They calculate the area from scale drawings using given measurements.</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>including using standard units, square centimetres (cm²) and square metres (m²) and estimate the area of irregular shapes</p> <ul style="list-style-type: none"> estimate volume (e.g. using 1 cm³ blocks to build cubes and cuboids) and capacity (e.g. using water) solve problems involving converting between units of time use all four operations to solve problems involving measure (e.g. length, mass, volume, money) using decimal notation including scaling. 	<p>Pupils use all four operations in problems involving time and money, including conversions (e.g. days to weeks, leaving the answer as weeks and days).</p>
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> identify 3-D shapes, including cubes and other cuboids, from 2-D representations know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles draw given angles, and measure them in degrees (°) identify: <ul style="list-style-type: none"> angles at a point and one whole turn (total 360°) angles at a point on a straight line and ½ a turn (total 180°) other multiples of 90° 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils become accurate in drawing lines with a ruler to the nearest millimetre, and measuring with a protractor. They use conventional markings for parallel lines and right angles.</p> <p>Pupils use the term diagonal and make conjectures about the angles formed by diagonals and sides, and other properties of quadrilaterals, for example using dynamic geometry ICT tools.</p> <p>Pupils use angle sum facts and other properties to make deductions about missing angles and relate these to missing number problems.</p>

Year 5 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ use the properties of rectangles to deduce related facts and find missing lengths and angles ▪ distinguish between regular and irregular polygons based on reasoning about equal sides and angles. 	
<p>Position and direction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed. 	<p>Position and direction</p> <p>Pupils recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and coordinates in the first quadrant. Reflection should be in lines that are parallel to the axes.</p>
<p>STATISTICS</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ solve comparison, sum and difference problems using information presented in a line graph ▪ complete, read and interpret information in tables, including timetables. 	<p>STATISTICS</p> <p>Pupils connect their work on coordinates and scales to their interpretation of time graphs.</p> <p>They begin to decide which representations of data are most appropriate and why.</p>

Year 6

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<p>NUMBER</p> <p>Number and place value</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ read, write, order and compare numbers up to 10 000 000 and determine the value of each digit▪ round any whole number to a required degree of accuracy▪ use negative numbers in context, and calculate intervals across zero▪ solve number and practical problems that involve all of the above.	<p>NUMBER</p> <p>Number and place value</p> <p>Pupils use the whole number system, including saying, reading and writing numbers accurately.</p>
<p>Addition, subtraction, multiplication and division</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none">▪ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication▪ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	<p>Addition, subtraction, multiplication and division</p> <p>Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Appendix 1).</p> <p>They undertake mental calculations with increasingly large numbers and more complex calculations.</p> <p>Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.</p>

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> perform mental calculations, including with mixed operations and large numbers. identify common factors, common multiples and prime numbers use their knowledge of the order of operations to carry out calculations involving the four operations solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. 	<p>Pupils round answers to a specified degree of accuracy, e.g. to the nearest 10, 20, 50 etc, but not to a specified number of significant figures.</p> <p>Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.</p> <p>Common factors can be related to finding equivalent fractions.</p>
<p>Fractions (including decimals and percentages)</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> use common factors to simplify fractions; use common multiples to express fractions in the same denomination compare and order fractions, including fractions > 1 add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$) 	<p>Fractions (including decimals and percentages)</p> <p>Pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (e.g. $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$) and progress to varied and increasingly complex problems.</p> <p>Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and</p>

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ divide proper fractions by whole numbers (e.g. $\frac{1}{3} \div 2 = \frac{1}{6}$) ▪ associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $\frac{3}{8}$) ▪ identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places ▪ multiply one-digit numbers with up to two decimal places by whole numbers ▪ use written division methods in cases where the answer has up to two decimal places ▪ solve problems which require answers to be rounded to specified degrees of accuracy ▪ recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. 	<p>as equal parts of objects, for example as parts of a rectangle.</p> <p>Pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (e.g. if $\frac{1}{4}$ of a length is 36cm, then the whole length is $36 \times 4 = 144\text{cm}$).</p> <p>They practise with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators.</p> <p>Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (e.g. $3 \div 8 = 0.375$). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context. Pupils multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the simplest cases, such as $0.4 \times 2 = 0.8$, and in practical contexts, such as measures and money.</p> <p>Pupils are introduced to the division of decimal numbers by one-digit whole numbers and, initially, in practical contexts involving measures and money. They recognise division calculations as the inverse of multiplication.</p> <p>Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their</p>

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
	<p>answers to decimal calculations. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.</p>
<p>RATIO AND PROPORTION</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts ▪ solve problems involving the calculation of percentages (e.g. of measures) such as 15% of 360 and the use of percentages for comparison ▪ solve problems involving similar shapes where the scale factor is known or can be found ▪ solve problems involving unequal sharing and grouping using knowledge of fractions and multiples. 	<p>RATIO AND PROPORTION</p> <p>Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (e.g. similar shapes, recipes).</p> <p>Pupils link percentages or 360° to calculating angles of pie charts.</p> <p>Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They might use the notation a:b to record their work.</p> <p>Pupils solve problems involving unequal quantities e.g. 'for every egg you need three spoonfuls of flour', '$\frac{3}{5}$ of the class are boys'. These problems are the foundation for later formal approaches to ratio and proportion.</p>
<p>ALGEBRA</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ express missing number problems algebraically ▪ use simple formulae expressed in words 	<p>ALGEBRA</p> <p>Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as:</p> <ul style="list-style-type: none"> ▪ missing numbers, lengths, coordinates and angles

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> ▪ generate and describe linear number sequences ▪ find pairs of numbers that satisfy number sentences involving two unknowns ▪ enumerate all possibilities of combinations of two variables. 	<ul style="list-style-type: none"> ▪ formulae in mathematics and science ▪ arithmetical rules (e.g. $a + b = b + a$) ▪ generalisations of number patterns ▪ number puzzles (e.g. what two numbers can add up to).
<p>MEASUREMENT</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ▪ solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate ▪ use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places ▪ convert between miles and kilometres ▪ recognise that shapes with the same areas can have different perimeters and vice versa ▪ recognise when it is possible to use formulae for area and volume of shapes ▪ calculate the area of parallelograms and triangles ▪ calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3), and extending to other units 	<p>MEASUREMENT</p> <p>They connect conversion (e.g. from kilometres to miles) to a graphical representation as preparation for understanding linear/proportional graphs.</p> <p>They know approximate conversions and are able to tell if an answer is sensible.</p> <p>Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.</p> <p>They relate the area of rectangles to parallelograms and triangles, e.g. by dissection, and calculate their areas, understanding and using the formulae to do this.</p> <p>Pupils could be introduced to compound units for speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.</p>

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
such as mm^3 and km^3 .	
<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> draw 2-D shapes using given dimensions and angles recognise, describe and build simple 3-D shapes, including making nets compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles. 	<p>GEOMETRY</p> <p>Properties of shapes</p> <p>Pupils draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.</p> <p>Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.</p> <p>These relationships might be expressed algebraically e.g. $d = 2 \times r$; $a = 180 - (b + c)$.</p>
<p>Position and direction</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> describe positions on the full coordinate grid (all four quadrants) 	<p>Position and direction</p> <p>Pupils draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers.</p>

Year 6 programme of study (statutory requirements)	Notes and guidance (non-statutory)
<ul style="list-style-type: none"> draw and translate simple shapes on the coordinate plane, and reflect them in the axes. 	<p>Pupils draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically e.g. translating vertex (a, b) to $(a-2, b+3)$; (a, b) and $(a+d, b+d)$ being opposite vertices of a square.</p>
<p>STATISTICS</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> interpret and construct pie charts and line graphs and use these to solve problems calculate and interpret the mean as an average. 	<p>STATISTICS</p> <p>Pupils connect their work on angles, fractions and percentages to the interpretation of pie charts.</p> <p>Pupils both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects.</p> <p>They should connect conversion from kilometres to miles in measurement to its graphical representation.</p> <p>Pupils know when it is appropriate to find the mean of a data set.</p>

Mathematics Appendix 1: Examples of formal written methods for multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes	874 - 523 becomes	932 - 457 becomes	951 - 427 becomes
$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline \end{array}$	$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \\ \hline \end{array}$	$\begin{array}{r} 932 \\ - 457 \\ \hline 475 \\ \hline \end{array}$	$\begin{array}{r} 951 \\ - 427 \\ \hline 524 \\ \hline \end{array}$
Answer: 1 431	Answer: 351	Answer: 475	Answer: 524

Short multiplication

24 × 6 becomes	24 × 12 becomes	124 × 12 becomes
$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ \times 12 \\ \hline 288 \\ \hline \end{array}$	$\begin{array}{r} 124 \\ \times 12 \\ \hline 1488 \\ \hline \end{array}$
Answer: 144	Answer: 288	Answer: 1 488

Long multiplication

24 × 16 becomes	24 × 32 becomes	124 × 26 becomes
$\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 240 \\ \hline 384 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ \times 32 \\ \hline 48 \\ 720 \\ \hline 768 \\ \hline \end{array}$	$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ \hline \end{array}$
Answer: 384	Answer: 768	Answer: 3 224

Short division

432 ÷ 5 becomes

$$\begin{array}{r} 86 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Answer: 86 remainder 2

574 ÷ 15 becomes

$$\begin{array}{r} 38 \\ 15 \overline{) 574} \\ \underline{45} \\ 124 \\ \underline{120} \\ 4 \end{array}$$

Answer: $38 \frac{4}{15}$

511 ÷ 35 becomes

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 161 \\ \underline{140} \\ 210 \\ \underline{210} \\ 0 \end{array}$$

Answer: 14.6

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

15×20

15×8

$$\frac{12}{15} = \frac{4}{5}$$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28.8